Calculus Warm Up # 7-2

No Grapher

Find any extrema and/or points of inflection and describe the concavity of the function.

$f(x) = 3x^4 + 4x^3$

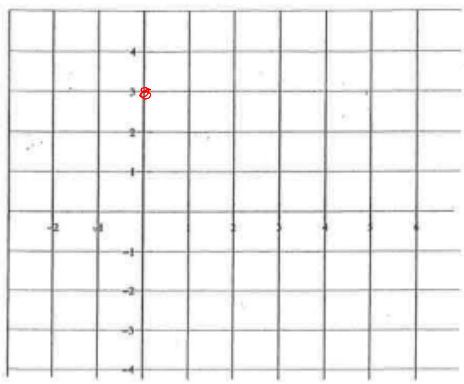
Turn in HW:

p. 197 # 1, 11, 23

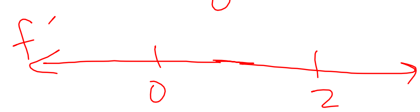
* Be thorough in your investigation of the function and accurately graph it on graph paper!
This assignment is due turned in tomorrow.

II. Draw a possible graph of a function f that has ALL of the following properties. Clearly indicate where f has local extrema and points of inflection.

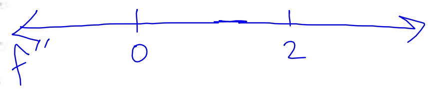
- For $x < 0$, $f'(x) > 0$ *and* $f''(x) > 0$
- $\boxed{f(0) = 3,}$ $f'(0) > 0$, $f''(0) = 0$
- For $0 < x < 2$, $f'(x) > 0$ *and* $f''(x) < 0$
- $f'(2) = 0$ possible extrema
- For $x > 2$, $f'(x) < 0$ *and* $f''(x) < 0$



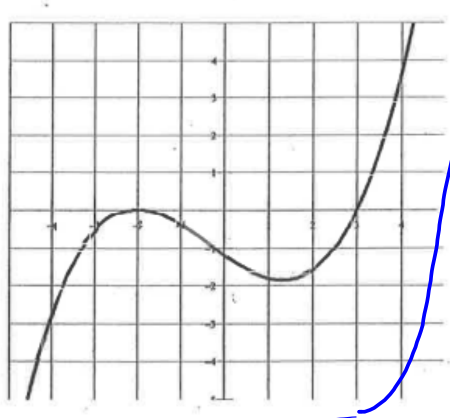
- Start with (0,3)
- look at f' for each side of that incr or decreasing.



- look @ f'' for PI's & concavity



III. Below is the graph of the ***second*** derivative of g. Use the graph to answer the following questions on the interval $[-4,4]$.



Graph of g''

(4) g' will have a minimum where the slopes of g' (that would be g'') go from negative to positive.

1. Where is g concave down? Concave up? Justify your answer.
2. Where does g have points of inflection? Justify your answer.
3. Suppose that $g'(0) = 0$. Is g increasing or decreasing at $x = 2$? Justify your answer.
4. Where does g' have a local minimum? Justify your answer.

4.7

-Optimization problems

Many applications call for finding things like:

greatest profit
least cost
least time
greatest strength
optimum size
least distance

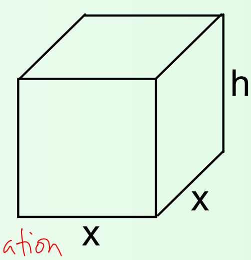
All are minimum or maximum values we can find with Calculus!

Ex: Design an open box with a square base and surface area = 108 sq. inches.

$108 = 4xh + x^2$

$h = \frac{108 - x^2}{4x}$

What are the dimensions that would maximize the volume of the box?



$V = x^2 h$

$= x^2\left(\frac{108 - x^2}{4x}\right) \longrightarrow$ simplify before differentiation

Plan to maximize volume:
Find V′, set = 0,
get critical #'s,
confirm max.

$V = 27x - \frac{x^3}{4}$

$V' = 27 - \frac{3}{4}x^2$

critical # $\rightarrow x = 6$

2nd deriv. test to confirm:

$V'' = -\frac{3}{2}x$

$V''(6) = -$ so concave down $\cap$

confirms max

$\boxed{\text{Dimensions are } 6 \text{ in} \times 6 \text{ in} \times 3 \text{ in.}}$ for max Volume

Steps:

1. Assign symbols for quantities given and to be determined, make a sketch if possible.

2. Write a primary equation for the quantity to be maximized or minimized. Consider its domain.

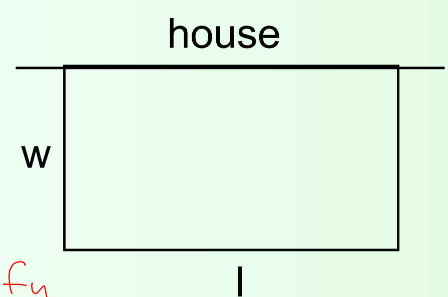
3. Reduce the primary equation to a single independent variable. (You might need a second relationship between the variables to do that.)

4. Take the derivative of the primary equation, find critical numbers, confirm extrema to answer the question.

Ex: For my garden, I want to enclose the largest rectangular area possible with 76m of fencing. I plan to use the house for one side of the fenced rectangle. What are the dimensions of the garden?

$\to 76 = 2w + l$

$l = 76 - 2w$



$A = lw$

$A = (76-2w)w$

simplify

Plan to maximize area:
Find A´, set = 0,
get critical #'s,
confirm max.

$A = 76w - 2w^2$

$A' = 76 - 4w$

critical # $w = 19$

confirm with A''

$A'' = -4$ always

concave down confirms Max

Dimensions: 19m × 38m

Find the points (x, y) on the graph of $y = 4 - x^2$ that are closest to the point (0, 2).

$x^2 = 4 - y$

Minimizing distance:
so primary equation is the distance formula!

Secondary relationship is given.

$\boxed{\left(\pm\ ,\ \frac{5}{2}\right)}$

$d = \sqrt{(x-0)^2 + (y-2)^2}$ ← minimum of this will lead to min outcome

$L = (x-0)^2 + (y-2)^2$

$L = x^2 + y^2 - 4y + 4$

$L = 4 - y + y^2 - 4y + 4$

$L = y^2 - 5y + 8$

$L'' = 2$ (+)

$L' = 2y - 5$ → confirms conc. up

$0 =$ Min @ $y = \frac{5}{2}$

$y = \frac{5}{2}$

HW: p. 203 # 5, 6, 9, 11, 13, 15, 21, 23

Group Quiz tomorrow:
Completely analyze and accurately sketch a graph.